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Stability analysis of the mechanism of jet attachment to walls

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Abstract

The attachment of fluid jets to walls when blown closed to these walls is known by the name of the Coanda effect and is widely used in practice, e.g., for the ventilation and air-conditioning of rooms. When a jet of air cooler than the ambient air is blown horizontally along the ceiling, the jet adheres to the ceiling over a given distance even though it is heavier than the ambient air. The aim of this paper is to provide a satisfactory explanation for the phenomenon, on the basis of a theoretical analysis of the equilibrium of a layer of fluid that is denser than the ambient fluid, when it is driven by a translation movement in relation to the ambient fluid and is located under a flat wall in a gravity field. The analysis makes it possible to formulate the separation distance of the jet depending on its properties at the blowing slot. That distance is then compared with the available experimental results.

The resultant formula, connecting the ratio between the separation distance and the height of the blowing slot and the Archimedes number of the jet, is as follows:

$$\frac{x_{\mathrm{S\,th}}}{a} = \frac{3.6}{Ar_0^{0.64}}.$$

The agreement between the results of the theory proposed here and the available experimental results in the literature bears out the validity of the theory.

1. Introduction

Fluid jets have a natural tendency to attach to walls when projected close to them. That property occurs even if a jet of air that is colder and therefore denser than the ambient air is blown along a flat wall located above the ambient air in a gravity field. If there is no wall (Fig. 1), the streamlines of the flow curve down as soon as it emerges from a plane slot and takes on an approximately parabolic form, in accordance with the laws of gravity.

On the other hand, if the jet is projected below a wall (Fig. 2), it stays attached to the wall and only moves down at a distance of x_S from its starting point. That distance depends on the height of the blowing slot, the temperature T_c and the velocity U_0 of the jet and the ambient temperature T_h.

Figs. 1 and 2 are the result of bi-dimensional numerical simulation of a planar turbulent jet from an opening that is level with the ceiling at a height of 0.02 m, at a horizontal velocity and initial temperature equal to 1.4 m s^{-1} and 10 °C, respectively, while the ambient atmosphere is at 20 °C. The positions of the jets are marked by equal temperature lines.

The apparently paradoxical phenomenon, initially described by Young [1] in 1800, was widely used in the early 20th century by Coanda in the machines he invented. That is why the effect has been given his name, even though there is no written trace of his works. Several experimenters [2–5] subsequently studied its effects, particularly for ventilation and air-conditioning applications for rooms. However, they did not explain its reason.

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Nomenclature

a	height of the blowing slot (m)
c_p	specific heat at constant pressure ($\mathrm{J\ kg^{-1}\ K^{-1}}$)
g	acceleration of gravity ($\mathrm{m\ s^{-2}}$)
h_m	mean height of the jet at coordinate x (m)
I_u	sum of $f(\eta)$: see formula (7)
I_{u^2}	sum of $f^2(\eta)$: see formula (9)
I_θ	sum of $h(\eta)$: see formula (22)
$I_{u\theta}$	sum of $h(\eta)f(\eta)$: see formula (7)
k_u	constant occurring in the law of velocity inside the jet
k_θ	constant occurring in the law of temperature inside the jet
L	width of the blowing slot (m)
p_i	pressure at point i (Pa)
$q_m(x)$	mass rate of flow in the jet at coordinates x ($\mathrm{kg\ s^{-1}}$)
$q_{mv}(x)$	momentum rate of flow in the jet at coordinates x ($\mathrm{m\ kg\ s^{-2}}$)
T_c	temperature of cold air at the slot (K)
T_h	temperature of hot ambient air (K)
$u(x,y)$	velocity inside the jet at coordinates (x,y) ($\mathrm{m\ s^{-1}}$)
$U_M(x)$	maximal velocity ($\mathrm{m\ s^{-1}}$)
U_0	velocity of the jet at the slot ($\mathrm{m\ s^{-1}}$)
V_m	mean velocity of the jet at coordinates x ($\mathrm{m\ s^{-1}}$)
x	horizontal distance from the slot (m)
x_s	separation distance of the jet (m)
y	vertical distance from the slot (m)
α	exponent of the velocity law
β	exponent of the temperature law
λ	coefficient of the growing law of the jet
$\eta = y/(\lambda x)$	reduced ordinate
$\rho(x,y)$	difference of density between a point of the jet and the ambient air ($\mathrm{kg\ m^{-3}}$)
ρ_c	density of the cold air at the slot ($\mathrm{kg\ m^{-3}}$)
ρ_h	density of the hot ambient air ($\mathrm{kg\ m^{-3}}$)
ρ_m	mean density of the air inside the jet at coordinate x ($\mathrm{kg\ m^{-3}}$)
$\theta(x,y)$	difference of temperature between a point of the jet and the ambient air (K)
$\Theta_M(x)$	maximum difference of temperature between the jet and the ambient air (K)
Θ_0	difference of temperature between the jet and the ambient air at the slot (K)
$Ar = (\Delta\rho_m g h_m)/(\rho_h V_m^2)$	Archimedes number of the jet at coordinate x
$Ar_0 = (ag\Theta_0)/(T_h U_0^2)$	Archimedes number of the jet at the blowing slot

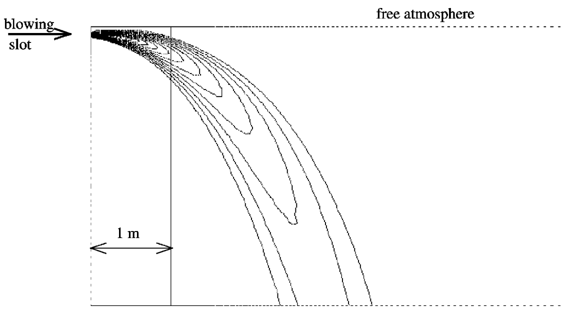

Fig. 1. Lines of equal temperature of a jet of cold gas in a free atmosphere.

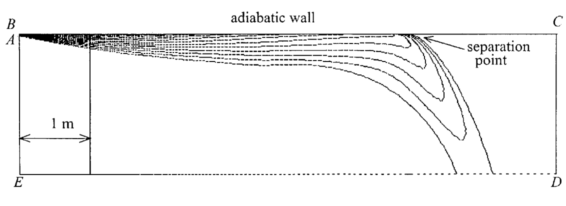

Fig. 2. Lines of equal temperature of a cold jet flowing under a flat adiabatic wall.

This paper is aimed at filling that gap. It is based on the study of the equilibrium of a layer of fluid that is denser than the ambient fluid and is in motion in relation to the ambient fluid. The study results in a theoretical formulation of the separation distance of the jet. After reviewing the classic properties of wall jets, from which the velocity and temperature distribution can be inferred, a theoretical analysis of the stability of the jet will be made, resulting in the expression of the separation distance. By comparing the law with experimental results, the validity of the theory is borne out.

2. Use of the knowledge of wall jets

The jets considered here are planar bi-dimensional wall jets from a blowing slot of height a, of infinite width, located just below a smooth wall (Fig. 3). These jets have been the subject of many theoretical and experimental studies. In a recapitulating book, Rajaratnam [6] provides the self-similar velocity and temperature profiles stated below, which will be used in the theoretical analysis in Section 3.

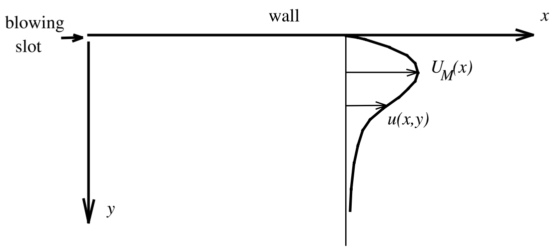

Fig. 3. Transverse velocity profile in a planar wall jet.

In an area located sufficiently downstream from the potential flow of the jet, if the transfers are to be dominated by turbulent diffusion ($x/a > 20$), the horizontal component of the velocity $u(x,y)$ represented schematically in Fig. 3 is

$$\frac{u(x,y)}{U_{\mathrm{M}}(x)} = f(\eta), \tag{1}$$

where the maximum velocity in the jet on abscissa x is

$$U_{\mathrm{M}}(x) = k_u \left(\frac{a}{x}\right)^{\alpha} U_0, \tag{2}$$

where k_u and α are constants, η designates a reduced ordinate: $\eta = y/(\lambda x)$. Because of the similarity between the velocity profiles, the value of λ is constant and function f is independent of the characteristics of the jet and distance x.

Likewise, when the fluid is blown at a temperature that is different from the ambient temperature, the temperature difference between a point of the jet and the ambient fluid is

$$\frac{\theta(x,y)}{\Theta_{\mathrm{M}}(x)} = h(\eta), \tag{3}$$

where the maximum temperature difference in the jet on abscissa x is

$$\Theta_{\mathrm{M}}(x) = k_\theta \left(\frac{a}{x}\right)^{\beta} \Theta_0. \tag{4}$$

As before, k_θ and β are constants.

Assuming that air obeys the law of perfect gases, using an asymptotic expansion to order 1 in Θ_0/T_{h}, the density at any point of the jet is

$$\rho(x,y) = \rho_{\mathrm{h}}\left[1 + \frac{\Theta_0}{T_{\mathrm{h}}} k_\theta \left(\frac{a}{x}\right)^{\beta} h(\eta)\right]. \tag{5}$$

The mass flow rate per unit width L on the abscissa x is then

$$\frac{q_{\mathrm{m}}(x)}{L} = \rho_{\mathrm{m}} h_{\mathrm{m}} V_{\mathrm{m}} = \int_0^{\infty} \rho(x,y) u(x,y)\,\mathrm{d}y$$
$$= \rho_{\mathrm{h}} \lambda k_u U_0 \left(\frac{a}{x}\right)^{\alpha} x \left[I_u + \frac{\Theta_0}{T_{\mathrm{h}}} k_\theta \left(\frac{a}{x}\right)^{\beta} I_{u\theta}\right], \tag{6}$$

where the following is given:

$$I_u = \int_0^{\infty} f(\eta)\,\mathrm{d}\eta \quad \text{and} \quad I_{u\theta} = \int_0^{\infty} f(\eta) h(\eta)\,\mathrm{d}\eta. \tag{7}$$

Integrals I_u and $I_{u\theta}$ are constants as a result of the independence of functions f and h from x and the properties of the jet.

The horizontal momentum flow rate, per unit width, is

$$\frac{q_{\mathrm{mv}}(x)}{L} = \rho_{\mathrm{m}} h_{\mathrm{m}} V_{\mathrm{m}}^2 = \int_0^{\infty} \rho(x,y) u^2(x,y)\,\mathrm{d}y$$
$$= \rho_{\mathrm{h}} \lambda k_u^2 U_0^2 \left(\frac{a}{x}\right)^{2\alpha} x \left[I_{u^2} + \frac{\Theta_0}{T_{\mathrm{h}}} k_\theta \left(\frac{a}{x}\right)^{\beta} I_{u^2\theta}\right]. \tag{8}$$

The integrals involved in this formula are defined by

$$I_{u^2} = \int_0^{\infty} f^2(\eta)\,\mathrm{d}\eta \quad \text{and} \quad I_{u^2\theta} = \int_0^{\infty} f^2(\eta) h(\eta)\,\mathrm{d}\eta. \tag{9}$$

Given that because of the force of friction at the ceiling, the momentum flow rate provided by relationship (8) can only decrease when x increases, α is necessarily greater than 0.5. The lower the loss of momentum flow rate due to that friction, the α is closer to 0.5.

Rajaratnam [6] and Albright and Scott [7] supply a number of numerical values for the constants used above, and also experimental expressions for functions f and h:

$$f(\eta) = 1.48\eta^{1/7}[1 - \mathrm{erf}(0.68\eta)], \tag{10}$$

$$h(\eta) = \exp[-0.8(\eta - 0.1)^{1,4}]$$
$$\text{if } \eta > 0,1; \qquad h(\eta) = 1 \quad \text{else.} \tag{11}$$

The data can also be obtained by the numerical integration of the momentum and enthalpy balance equations associated with a turbulence model representative of flows in jets. The method was applied with a k–ε turbulence model, which does not use any wall law but integrates the laminar boundary layer close to the walls. Subject to the constraint of a fine mesh in the region ($y^+ < 5$) that is involved by it, this model known as the *two-layer zonal model* [8,9] provides satisfactory precision for turbulent flows with a low Reynolds number. The upwind numerical scheme is used [10].

The domain was divided into 400 cells in the lengthwise direction and 110 in the vertical direction. A finer grid was used near the wall to follow the description of the laminar profile at the wall (first cell about 0.003 m thick).

The boundary conditions are:

- at the jet blowing slot (AB in Fig. 2): constant velocity and temperature (10 K below the ambient air in all simulations);
- at the vertical wall with the blowing slot (AE in Fig. 2) and horizontal ceiling (BC in Fig. 2) considered to be smooth: zero velocity and adiabatic walls;

Table 1
Calculation of numerical values of jet constants

	Jet 1	Jet 2	Jet 3	Jet 4	Jet 5	Jets average	Value given by [6] or [7]
a	0.02	0.02	0.05	0.01	0.05		
U_0	1.4	2.0	1.4	2.5	1.0		
Θ_0	10	10	10	10	10		
α	0.510	0.510	0.505	0.520	0.500	0.51	0.5
β	0.440	0.450	0.460	0.440	0.450	0.45	None
k_u	3.43	3.54	3.56	3.55	3.39	3.5	3.5
k_θ	3.38	3.55	3.69	3.46	3.42	3.5	None
λ	0.108	0.100	0.104	0.099	0.108	0.10	0.068
I_u	1.093	1.071	1.094	1.036	1.133	1.09	1.09
I_{u^2}	0.761	0.747	0.756	0.724	0.783	0.76	0.75
I_θ	1.160	1.146	1.125	1.119	1.166	1.14	1.17
$I_{u\theta}$	0.714	0.708	0.705	0.685	0.728	0.71	0.75
$I_{u^2\theta}$	0.544	0.540	0.542	0.520	0.558	0.54	0.59

- the other limits (CDE in Fig. 2) represent communication with an infinite environment at the ambient temperature: if the fluid leaves the calculation domain, the relative static pressure (gravity contribution not included) is imposed to a zero value; if the fluid enters, the total pressure is imposed to a zero value and the temperature is equal to the ambient temperature.

Table 1 contains the features of the five jets studied and the values of the constants resulting from this method, which may be compared with those inferred from reference (works) [6] and [7], directly or by means of numerical integrations based on formulae (7), (9)–(11).

It can be seen that apart from λ, when constants are published by the experimenters, they do not vary much from those obtained using the calculation.

Figs. 4 and 5 show the high quality of the correlations (2) and (4) selected for the maximum velocity and temperature variations relating to the five jets studied, with different distances x.

As the bibliographical data contained in [6] and [7] do not provide sufficiently complete results, the average values from the simulations given in the last column but one of Table 1 were used for what follows.

3. Jet attachment stability theory

The main hypothesis of the theory developed below is that the velocity and temperature profiles are self-similar as presented here above. This must be considered as a very good hypothesis as it was observed by many experimentators. If a layer of immobile cool air is located above a mass of warmer air, that mass of air cannot enjoy a stable balance. That may seem obvious, but its demonstration will act as a guide for the reasoning outlined below in respect of jet attachment.

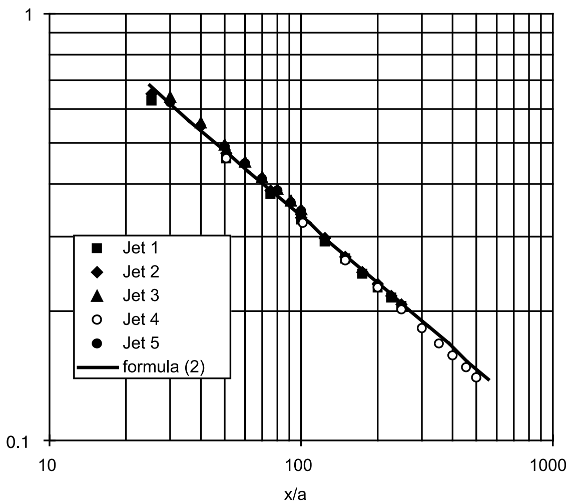

Fig. 4. Maximum velocity correlation.

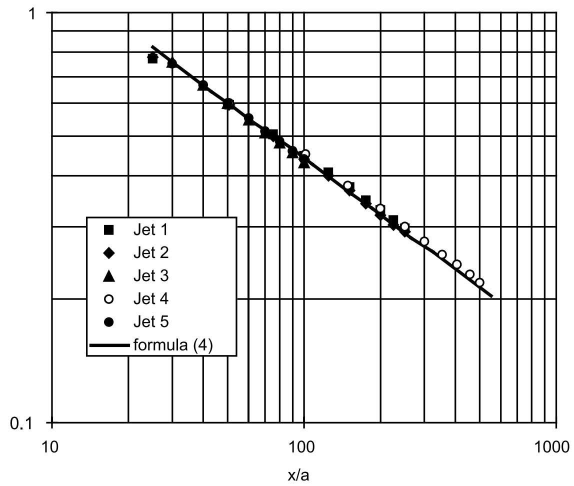

Fig. 5. Maximum temperature difference correlation.

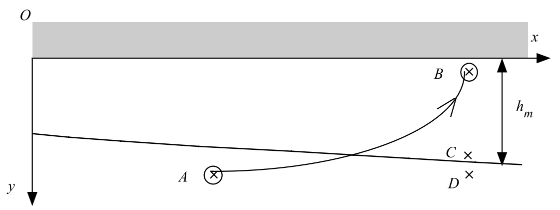

Fig. 6. Schematic chart for the demonstration.

As shown in Fig. 6, let us suppose an immobile layer of air ($U_0 = 0$) with a density of ρ_c located just beneath the ceiling of an enclosure filled with warmer air, with a density ρ_h that is therefore lower.

A small perturbation may occur in any place (point A in Fig. 6) whereby a bubble of lighter air enters into the colder air. Because of its buoyancy, the air bubble will rise up to the ceiling. The time taken to transfer that air particle is very short compared to its thermal time constant. It may therefore be assumed that it retains its temperature and therefore its density.

If losses due to the friction of the particle during its transfer from A to B are ignored, the conservation of its mechanical energy is expressed as follows:

$$p_B = p_A - \rho_h g h_m. \tag{12}$$

Now, if the law of the static of fluids is applied between points B and C in the cold air, the result is

$$p_C = p_B + \rho_c g h_m. \tag{13}$$

As points A and D are on the same horizontal, they have the same pressure. The following can then be inferred from (12) and (13):

$$p_C - p_D = (\rho_c - \rho_h) g h_m > 0. \tag{14}$$

In this way, a vertical downward force at the interface between the cold (point C) and hot air (point D) leads to a drop in the cooler layer of air with an unstable equilibrium. This is evident, but as it shall be shown, that equilibrium can become stable if the layer of air has a certain velocity.

If the layer of air (jet parallel to the ceiling) moves horizontally, the phenomenon of the transfer of a particle of hot air can occur here too. Let us assume that there is such a transferred air bubble, whose size is sufficient for its temperature and therefore its density to remain unchanged during its move from A to B.

The separation between the air in the jet and the ambient air is somewhat artificial. The jet thickness h_m is defined precisely with formula (18), where the values of ρ_m and V_m are defined by formulae (6) and (8).

Let Δp be the volume of mechanical energy loss of the bubble for its travel from A to B. It is assumed that the driving effect of the jet during that travel over the entire thickness of the jet gives the particle mass a kinetic energy equal to the mean mass kinetic energy of the jet. The mechanical energy balance of the bubble during its move from A to B is therefore

$$(p_A - \rho_h g h_m) - \left(p_B + \rho_h \frac{V_m^2}{2}\right) = \Delta p. \tag{15}$$

If the current lines are assumed to be straight and parallel to each other, the equation of the static of fluids continues to apply between B and C, and may be written as

$$p_C - p_B = \int_0^{h_m} \rho g \, dy. \tag{16}$$

As points A and D are on the same horizontal, they have the same pressure. The following may be inferred from (15) and (16):

$$p_C - p_D = (\rho_m - \rho_h) g h_m - \rho_h \frac{V_m^2}{2} - \Delta p, \tag{17}$$

where the mean density is defined by the formula

$$\Delta\rho_m g h_m = (\rho_m - \rho_h) g h_m = g \int_0^{h_m} (\rho - \rho_h) \, dy \approx g \int_0^{\infty} (\rho - \rho_h) \, dy. \tag{18}$$

The jet will be stable if the pressure difference at the interface $p_C - p_D$ is negative, therefore creating an upward force. The stability criterion for the jet on abscissa x can be inferred to be

$$Ar = \frac{(\rho_m - \rho_h) g h_m}{\rho_h V_m^2} = \frac{\Delta\rho_m g h_m}{\rho_h V_m^2} \leqslant \frac{1}{2} + \frac{\Delta p}{\rho_h V_m^2}. \tag{19}$$

A sufficient condition for jet stability is that the Archimedes number is less than 1/2, because the loss of pressure Δp is always positive.

The jet remains stable as long as the Archimedes number is less than 1/2, if the loss of pressure due to friction is neglected. When the Archimedes number is 1/2, that provides a slightly underestimated value for the abscissa x_s of the fall of the jet in the ambient air.

In view of Eq. (5)

$$\Delta\rho = \rho(x, y) - \rho_h = \rho_h \frac{\Theta_M(x)}{T_h} h(\eta), \tag{20}$$

i.e., after integration

$$\frac{h_m \Delta\rho_m}{\rho_h} = \frac{1}{\rho_h} \int_0^{\infty} \Delta\rho \, dy = k_\theta \frac{\Theta_0}{T_h} \left(\frac{a}{x}\right)^{\beta} I_\theta \lambda x, \tag{21}$$

where

$$I_\theta = \int_0^{\infty} h(\eta) \, d\eta. \tag{22}$$

The ratio of formulae (8) and (6) provides the average velocity

$$V_{\mathrm{m}} = k_u U_0 \left(\frac{a}{x}\right)^{\alpha} \frac{I_{u^2} + k_\theta(\Theta_0/T_{\mathrm{h}})(a/x)^{\beta} I_{u^2\theta}}{I_u + k_\theta(\Theta_0/T_{\mathrm{h}})(a/x)^{\beta} I_{u\theta}}. \tag{23}$$

Formulae (23) and (21) put into inequality (19), which is changed into equality and in which the loss of pressure Δp is ignored, provide the jet separation distance x_{s}

$$\frac{x_{\mathrm{s}}}{a} = \left(\frac{K}{Ar_0}\right)^{1/(1-\beta+2\alpha)} \tag{24}$$

the initial Archimedes number of the jet being defined by

$$Ar_0 = \frac{ag\Theta_0}{T_{\mathrm{h}} U_0^2} \tag{25}$$

and the value of K being

$$K = \frac{k_u^2}{2\lambda I_\theta k_\theta} \left[\frac{I_{u^2} + k_\theta(\Theta_0/T_{\mathrm{h}})(a/x)^{\beta} I_{u^2\theta}}{I_u + k_\theta(\Theta_0/T_{\mathrm{h}})(a/x)^{\beta} I_{u\theta}}\right]^2. \tag{26}$$

The value of K is in principle dependent on distance x_{s}. For the temperature variations encountered, for instance, in the field of air-conditioning, systematic numerical testing has shown that the Θ_0 terms in the numerator and denominator in the expression of K given to the second part of formula (26) are negligible.

That is due to the low difference between formulae (26) and (27): 2% for a value of x_{s}/a of 50, which may be considered as the lower applicability limit of the formula, and Θ_0/T_{h} of 10% which is, in air-conditioning, the extreme temperature difference between the blown air and the ambient air.

One could therefore say

$$K = \frac{k_u^2}{2\lambda I_\theta k_\theta} \left(\frac{I_{u^2}}{I_u}\right)^2. \tag{27}$$

4. Comparison with experimental results

Different authors have published experimental results about the separation distance of anisothermal wall jets in a gravity field, particularly Sandberg et al. [5], who have been selected for the comparison with the previous result, as they specify all their operating conditions. For practical reasons, their experiment did not involve an infinite planar jet, but a jet with a depth limited between two parallel planes at a distance of 0.2 m from each other, which is considerably less than the separation distances observed. The resulting increase in friction may lead to a reduction in the velocity, and therefore give rise to an underestimation of the separation distances. For Archimedes number values at the blowing slot located between 0.001 and 0.01, the following relationship may be inferred from these measurements between that number and the separation distance:

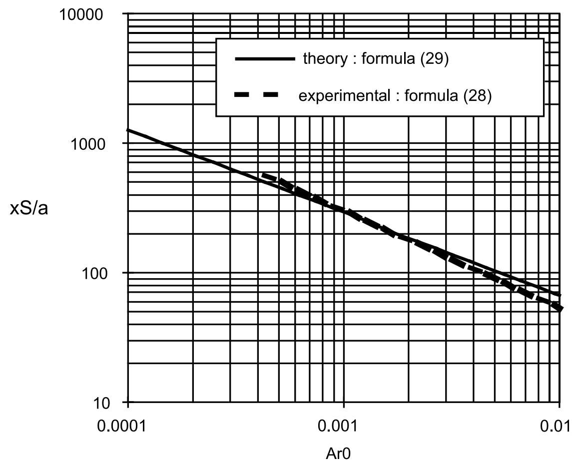

Fig. 7. Comparison between theory and experiment.

$$\frac{x_{\mathrm{S\,exp}}}{a} = \frac{1.7}{Ar_0^{0.75}}. \tag{28}$$

The application of the numerical values of the constants established in Table 1 to formulae (23) and (26) provides the following:

$$\frac{x_{\mathrm{S\,th}}}{a} = \frac{3.6}{Ar_0^{0.64}}. \tag{29}$$

As shown by Fig. 7, the two formulae (28) and (29) provide very close results for the Archimedes number interval in which the measurements are located – the difference is practically nil for $Ar_0 = 0.001$ and is 20% when $Ar_0 = 0.01$, which is the limit of validity of the experimental relationship. The fact that the experimental value tends to be lower than the theoretical value is consistent with the remark about the possible underestimation of the measured separation distance.

5. Conclusion

The theory presented is based on the analysis of the stability of the equilibrium of a layer of gas moving along a wall and subjected to a force tending to separate it from the wall because the layer is made up of gas that is heavier than the surrounding air. A formula (29) was obtained to link the ratio between the jet separation distance and the height of the discharge opening and the initial Archimedes number of the jet. The only hypothesis of the theory is that the velocity and temperature profiles are self-similar which must be considered as a very good hypothesis as it was observed by many experimentators.

The formula obtained in this way provides results that agree with the experimentation of Sandberg et al.

[5], especially in respect of low Archimedes number values.

Provided that the basic hypothesis of self-similar profile of velocity and temperature is satisfied, the theory presented above can be extended to other cases, e.g., that of the flow of a jet with a constant density along a convex wall, where gravity is replaced by the centrifugal force. The separation of a jet from a cylindrical wall can then be obtained using an approach similar to that described here.

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